

MATEMATIKA – I RAZRED

23.3.2020. – 27.3.2020.

Nastavna jedinka: Potencije s racionalnim eksponentom. Račun s korijenima.

Pozdrav svima, vraćamo se online nastavi..

Vrijeme je za nastavak rada s korijenima. Tijekom prošle sedmice postavljeni su vam materijali iz lekcije „Račun s korijenima“, (Zadaću nisu predali Mario Gavran, Antonio Batarilo, Toni Martić, Marin Kožul i Marija Šaravanja – te će u skladu s tim biti i vrednovani), tako da bi što bolje razumjeli računanje s korijenima potrebno je da ponovo pročitate materijale od prošle sedmice, kao i ono što smo radili u školi o racionalizaciji nazivnika.

U prilogu vam šaljem materijal iz lekcije „Potencije s racionalnim eksponentom“ koji imate i u vašim udžbenicima, kao i formule s korijenima i potencijama na jednom mjestu. Također možete posjetiti i priloženu stranicu kako biste bolje utvrdili gradivo, te pogledati priložene videomaterijale.

https://edutorij.e-skole.hr/share/proxy/alfresco-noauth/edutorij/api/proxy-guest/20d452c4-8951-42b8-9314-263ea1edf90d/html/278_Potencije_s_racionalnim_eksponentima.html

<https://www.mim-sraga.com/formule/korijeni.htm>

Zadaću ću vam okačiti kasnije kao assingment na Edmodu, a u ovoj zadaći ćemo ponoviti i racionalizaciju nazivnika.

Videomaterijali:

1. <https://www.youtube.com/watch?v=Vs8id1RxmB0>
2. https://www.youtube.com/watch?v=F_XY945RNUQ
3. <https://www.youtube.com/watch?v=ZaJ2LP2lsf0>
4. <https://www.youtube.com/watch?v=wVYLw7yg5JA>
5. <https://www.youtube.com/watch?v=iCOq5Se-jWw>
6. https://www.youtube.com/watch?v=l83VMbO_0KI
7. <https://www.youtube.com/watch?v=HmZFjubQ80O>
8. <https://www.youtube.com/watch?v=xvLtGldhyuo>
9. <https://www.youtube.com/watch?v=Fl2znb1J2pE>

Ako je a pozitivan realni broj i m, n prirodni brojevi, onda je

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}.$$

Izračunajmo:

1) $8^{-1/3}$; 2) $125^{-1/3}$; 3) $\left(\frac{1}{64}\right)^{-1/3}$; 4) $16^{-3/5}$.

1) $8^{-1/3} = \frac{1}{8^{1/3}} = \frac{1}{2}$. Pri potenciranju negativnim eksponentom najprije se rješavamo negativnog predznaka u eksponentu.

2) $125^{-1/3} = \frac{1}{125^{1/3}} = \frac{1}{5}$.

3) $\left(\frac{1}{64}\right)^{-1/3} = 64^{1/3} = 4$. Promjena predznaka u eksponentu mijenja bazu u njoj recipročnu.

4) $16^{-3/5} = \frac{1}{16^{3/5}} = \frac{1}{(16^{1/5})^3} = \frac{1}{(1.74110\dots)^3} = \frac{1}{5.27803\dots} = 0.18946\dots$.

Dakako, ovaj smo račun na džepnom računalu mogli i brže napraviti: $16^{-3/5} = 16^{-0.6} = 0.18946\dots$. Uvjerite se u to!

Zadatak 2. Izračunaj:

1) $27^{-2/3}$; 2) $\left(\frac{1}{16}\right)^{-1/4}$; 3) $625^{-1/5}$.

■ Računske operacije s potencijama

Za potencije s racionalnim eksponentom vrijede sva pravila koja smo utvrdili kod potencija s cjelobrojnim eksponentima.

Pravila potenciranja

Ako su a i b pozitivni brojevi, onda za sve racionalne brojeve r i s vrijedi

$$a^r \cdot a^s = a^{r+s}, \quad a^r : a^s = a^{r-s}, \quad (a^r)^s = a^{rs},$$

$$a^r \cdot b^r = (ab)^r, \quad \left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}.$$

Racionalizacija nazivnika kod korijena	
1.	$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \dots$
2.	$\frac{a}{\sqrt{a+\sqrt{b}}} = \frac{a}{\sqrt{a+\sqrt{b}}} \cdot \frac{\sqrt{a-\sqrt{b}}}{\sqrt{a-\sqrt{b}}} = \dots$
3.	$\frac{a}{\sqrt{a-\sqrt{b}}} = \frac{a}{\sqrt{a-\sqrt{b}}} \cdot \frac{\sqrt{a+\sqrt{b}}}{\sqrt{a+\sqrt{b}}} = \dots$
4.	$\frac{a}{\sqrt{a-\sqrt{b}}} = \frac{a}{\sqrt{a-\sqrt{b}}} \cdot \frac{\sqrt{a-\sqrt{b}}}{\sqrt{a-\sqrt{b}}} = \dots$
5.	$\frac{a}{\sqrt{a+\sqrt{b}}} = \frac{a}{\sqrt{a+\sqrt{b}}} \cdot \frac{\sqrt{a-\sqrt{b}}}{\sqrt{a-\sqrt{b}}} = \dots$
6.	$\frac{a}{\sqrt{\sqrt{a+\sqrt{b}}}} = \frac{a}{\sqrt{\sqrt{a+\sqrt{b}}}} \cdot \frac{\sqrt{\sqrt{a+\sqrt{b}}}}{\sqrt{\sqrt{a+\sqrt{b}}}} = \dots$
	$\frac{a}{\sqrt{\sqrt{a-\sqrt{b}}}} = \frac{a}{\sqrt{\sqrt{a-\sqrt{b}}}} \cdot \frac{\sqrt{\sqrt{a-\sqrt{b}}}}{\sqrt{\sqrt{a-\sqrt{b}}}} = \dots$
7.	$\frac{a}{\sqrt{\sqrt{a-\sqrt{b}}}} = \frac{a}{\sqrt{\sqrt{a-\sqrt{b}}}} \cdot \frac{\sqrt{\sqrt{a-\sqrt{b}}}}{\sqrt{\sqrt{a-\sqrt{b}}}} = \dots$
	$\frac{a}{\sqrt{\sqrt{a-\sqrt{b}}}} = \frac{a}{\sqrt{\sqrt{a-\sqrt{b}}}} \cdot \frac{\sqrt{\sqrt{a+\sqrt{b}}}}{\sqrt{\sqrt{a+\sqrt{b}}}} = \dots$
8.	$\frac{1}{\sqrt[3]{a \pm \sqrt[3]{b}}} = \frac{1}{\sqrt[3]{a \pm \sqrt[3]{b}}} \cdot \frac{\sqrt[3]{a^2 \mp \sqrt[3]{ab} + \sqrt[3]{b^2}}{\sqrt[3]{a^2 \mp \sqrt[3]{ab} + \sqrt[3]{b^2}}} = \dots$
9.	$\frac{1}{\sqrt[3]{a^2 \pm \sqrt[3]{ab} + \sqrt[3]{b^2}}} = \frac{1}{\sqrt[3]{a^2 \pm \sqrt[3]{ab} + \sqrt[3]{b^2}}} \cdot \frac{\sqrt[3]{a \mp \sqrt[3]{b}}}{\sqrt[3]{a \mp \sqrt[3]{b}}} = \dots$

Tetivni četverokut

$c \cdot f = a \cdot c + b \cdot d$
 $\alpha + \gamma = \beta + \delta = 180^\circ$
 $P = \sqrt{(s-a)(s-b)(s-c)(s-d)}$
 $o = a + b + c + d = 2s$

Tangencijalni četverokut

$P = \rho \cdot s = \frac{o}{2} \cdot \rho$
 $o = a + b + c + d = 2s$
 $a + c = b + d$

x^2	
$1^2 = 1$	$10^2 = 100$
$2^2 = 4$	$11^2 = 121$
$3^2 = 9$	$12^2 = 144$
$4^2 = 16$	$13^2 = 169$
$5^2 = 25$	$14^2 = 196$
$6^2 = 36$	$15^2 = 225$
$7^2 = 49$	$16^2 = 256$
$8^2 = 64$	$(-1)^2 = 1$
$9^2 = 81$	$(-2)^2 = 4$
x^3	
$1^3 = 1$	$8^3 = 512$
$2^3 = 8$	$9^3 = 729$
$3^3 = 27$	$10^3 = 1000$
$4^3 = 64$	$11^3 = 1331$
$5^3 = 125$	$(-1)^3 = -1$
$6^3 = 216$	$(-2)^3 = -8$
$7^3 = 343$	$(-3)^3 = -27$
x^4	
$1^4 = 1$	$1^5 = 1$
$2^4 = 16$	$2^5 = 32$
$3^4 = 81$	$3^5 = 243$
$4^4 = 256$	$4^5 = 1024$
$5^4 = 625$	$5^5 = 3125$
$6^4 = 1296$	$6^5 = 7776$
$(-1)^4 = 1$	$(-1)^5 = -1$
$(-2)^4 = 16$	$(-2)^5 = -32$
x^6	
$1^6 = 1$	$1^7 = 1$
$2^6 = 64$	$2^7 = 128$
$3^6 = 729$	$3^7 = 2187$
$4^6 = 4096$	$4^7 = 16384$
$5^6 = 15625$	$5^7 = 78125$
$6^6 = 46656$	$6^7 = 279936$
$(-1)^6 = 1$	$(-1)^7 = -1$
$(-2)^6 = 64$	$(-2)^7 = -128$

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MATEMATIČKE FORMULE ZA PRVI RAZRED SREDNJE ŠKOLE

ALGEBARSKI IZRAZI	POTENCIJE
$(a+b)^2 = (a+b) \cdot (a+b) = a^2 + 2ab + b^2$	$a^n \cdot a^m = a^{n+m}$
$(a+b)^2 = (b+a)^2$	$a^n : a^m = a^{n-m}$
$(a-b)^2 = (a-b) \cdot (a-b) = a^2 - 2ab + b^2$	$\frac{a^n}{a^m} = a^n : a^m = a^{n-m}$
$(a-b)^2 = (b-a)^2$	$(abc)^n = a^n b^n c^n$
$(-a-b)^2 = (a+b)^2$	$(a^n)^m = a^{n \cdot m}$
$(a-b) \cdot (a+b) = a^2 - b^2$	$((a^n)^m)^z = a^{n \cdot m \cdot z}$
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$
$a^3 - b^3 = (a-b) \cdot (a^2 + ab + b^2)$	$a^0 = 1$
$a^3 + b^3 = (a+b) \cdot (a^2 - ab + b^2)$	$a^1 = a$
$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$	$a^{-1} = \frac{1}{a}$
$x^2 + px + q = \left\{ \begin{matrix} m+n=p \\ m \cdot n=q \end{matrix} \right\} = (x+m) \cdot (x+n)$	$a^{-n} = \frac{1}{a^n}$
$ax^2 + bx + c = \left\{ \begin{matrix} m+n=b \\ m \cdot n=a \cdot c \end{matrix} \right\} = ax^2 + mx + nx + c = \dots$	
$a^{\frac{1}{n}} = \sqrt[n]{a}$	KORIJENI
$a^{-\frac{1}{n}} = \frac{1}{a^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{a}}$	$\sqrt[n]{\sqrt[n]{a}} = \sqrt[n \cdot n]{a}$
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$(\sqrt[n]{a^m})^p = \sqrt[n]{a^{m \cdot p}}$
$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{\sqrt[n]{a^m}}$	$(\sqrt[n]{a})^m = \sqrt[n]{a^m}$
$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
$\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$	$\sqrt[n]{a^n} = \sqrt[n]{a^n} = a$
	$2\sqrt[n]{a} = \pm \sqrt[2n]{a}$
	$\sqrt[n]{a^n} = \sqrt[m \cdot n]{a^{n \cdot m}}$
	$2^{n+1}\sqrt[n]{-a} = -2^{n+1}\sqrt[n]{a}$
	$\sqrt[n]{a^n} = \sqrt[m \cdot n]{a^{n \cdot m}}$
	$\sqrt[n]{0} = 0, \sqrt[n]{1} = 1$
	$\sqrt[n]{a^{n \cdot p} b} = a^p \cdot \sqrt[n]{b}$

7. Izračunaj:

$$1) \frac{10^{\frac{1}{5}} \cdot 2^{-0.6}}{5^{-1.4}};$$

$$2) \frac{15^{\frac{2}{3}} \cdot 3^{\frac{1}{3}}}{5^{-\frac{1}{3}}};$$

$$3) \frac{12^{\frac{3}{4}} \cdot 2^{-0.5}}{3^{-\frac{1}{4}}};$$

$$4) \frac{12^{-\frac{1}{3}} \cdot 2^{\frac{2}{3}}}{6^{-\frac{1}{3}} \cdot 4^{\frac{2}{3}}}.$$

8. Izračunaj vrijednost brojevnog izraza

$$\left[\left(a^{-\frac{2}{3}} b^{\frac{1}{3}} \right)^{0.5} : \left(a^{-\frac{3}{4}} b^{-\frac{3}{4}} \right)^{\frac{2}{3}} \right]^{-\frac{3}{2}},$$

$$\text{za } a = \frac{1}{16}, b = 2.$$

9. Izračunaj vrijednost brojevnog izraza

$$\left[\left(a^2 b^{-1.5} \right)^{-\frac{1}{3}} : \left(a^{-\frac{2}{3}} b \right)^2 \right]^{-\frac{1}{2}},$$

$$\text{za } a = \frac{8}{27}, b = \frac{1}{81}.$$

10. Izračunaj vrijednost brojevnog izraza

$$\left[\left(a^{-\frac{1}{4}} b^{\frac{1}{4}} \right)^{-2} : \left(a^2 b^{-1.5} \right)^{-\frac{1}{3}} \right]^{-\frac{1}{3}},$$

$$\text{za } a = 8, b = \frac{1}{64}.$$



18. Racionaliziraj nazivnik u razlomku:

1) $\frac{2}{\sqrt{6}}$;

2) $\frac{6}{\sqrt{3}\sqrt{5}}$;

3) $\sqrt{\frac{3}{4}}$;

4) $\frac{3\sqrt{2}}{2\sqrt{3}}$;

5) $\frac{a\sqrt{b}}{\sqrt{a}}$;

6) $\sqrt{\frac{2a}{3b}}$;

7) $\frac{2ab}{3\sqrt{6a}}$;

8) $\frac{2\sqrt{x} + 1}{3\sqrt{x}}$.

19. Racionaliziraj nazivnik u razlomku:

1) $\frac{\sqrt{3}}{2\sqrt{3} - 3}$;

2) $\frac{2\sqrt{5} - 3\sqrt{2}}{2\sqrt{5} + 3\sqrt{2}}$.

20. Racionaliziraj nazivnik u razlomku:

1) $\frac{36}{(3\sqrt{2} - 2\sqrt{3})^2}$;

2) $\frac{4}{(2\sqrt{5} - 3\sqrt{2})^2}$;

3) $\frac{\sqrt{3\sqrt{2} + \sqrt{3}}}{\sqrt{3\sqrt{2} - \sqrt{3}}}$;

4) $\frac{\sqrt{7 - 4\sqrt{3}}}{\sqrt{7 + 4\sqrt{3}}}$.

Zadaci 7.2.

1. Zapiši u obliku potencije:

1) $\sqrt[3]{4}$; 2) $\sqrt[4]{27}$; 3) $\frac{1}{\sqrt{10}}$; 4) $\sqrt[5]{16}$.

2. Izračunaj:

1) $16^{-\frac{1}{2}} - 0.25^{1.5}$; 2) $0.008^{-\frac{2}{3}} \cdot \left(\frac{1}{25}\right)^{-0.5}$;

3) $0.04^{-1.5} \cdot \left(\frac{1}{125}\right)^{\frac{2}{3}}$; 4) $\left(\frac{1}{9}\right)^{0.5} + 0.027^{-\frac{2}{3}}$.

3. Izračunaj:

1) $0.25^{-\frac{2}{3}} \cdot \left(\frac{1}{16}\right)^{-0.5}$; 2) $\left(\frac{1}{8}\right)^{-\frac{2}{3}} \cdot (0.81)^{-0.5}$;

3) $16^{0.5} + \left(\frac{1}{16}\right)^{-0.75}$; 4) $\left(\frac{27}{8}\right)^{-\frac{2}{3}} - 1.44^{-\frac{1}{2}}$.

4. Izračunaj:

1) $0.25^{-\frac{2}{3}} \cdot \left(\frac{1}{16}\right)^{-0.5}$; 2) $\left(\frac{1}{8}\right)^{-\frac{2}{3}} \cdot (0.81)^{-0.5}$;

3) $16^{0.5} + \left(\frac{1}{16}\right)^{-0.75}$; 4) $\left(\frac{27}{8}\right)^{-\frac{2}{3}} - 1.44^{-\frac{1}{2}}$.

5. Koliko je:

1) $\left(9^{-\frac{1}{2}} + (3\sqrt{3})^{-\frac{2}{3}}\right) \cdot \left(9^{-\frac{1}{2}} - (3\sqrt{3})^{-\frac{2}{3}}\right)$;

2) $\left(16^{-0.25} - (2\sqrt{2})^{\frac{1}{3}}\right) \left(16^{-0.25} + (2\sqrt{2})^{\frac{1}{3}}\right)$;

3) $\left(\sqrt[4]{18} - 2^{-\frac{1}{4}}\right) \left(\sqrt[4]{18} + 2^{-\frac{1}{4}}\right)$;

4) $\left[\left(27^{-\frac{1}{3}}\right)^{-\frac{2}{3}} - (\sqrt{125})^{\frac{1}{3}}\right] \cdot \left[\left(9^{\frac{1}{3}}\right)^{0.75} - (0.04)^{-\frac{1}{3}}\right]$